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Photoelastic Studies on Mixed Alum [$(NH_4)_{1-x}K_xAl(SO_4)_2.12H_2O$] Crystals

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Abstract

Stress–optical constants $(q_{11}-q_{12})$, $(q_{11}-q_{13})$ and q_{44} of mixed alum [$(NH_4)_{1-x}K_xAl(SO_4)_2.12H_2O$] crystals have been determined employing a modified form of Filon's spectrometer method in the wavelength range 4800–6200 Å. Corresponding strain–optical constants $(p_{11}-p_{12})$, $(p_{11}-p_{13})$ and p_{44} are derived from these using elastic constant data. It is observed that at a given wavelength all the constants vary regularly, though nonlinearly, with the composition of crystals. The deviation from linearity is found to be largest around equimolar concentrations.

1. Introduction

Study of various physical properties of mixed crystals, such as microhardness, dislocation density, dielectric constant, lattice constants, elastic and photoelastic constants, has been the subject of deep interest in recent years. Photoelasticity is the least studied aspect of mixed crystals. The investigations made so far on stress-induced birefringence have been confined to pure cubic crystals. Ethiraj (1975) studied the piezo-optic birefringence in (KCl–KBr) mixed crystals in the visible and ultraviolet regions.

Ammonium alum [$NH_4Al(SO_4)_2.12H_2O$] and potassium alum [$KAl(SO_4)_2 12H_2O$] belong to the T_h class of the cubic system, characterized by four independent stress–optical constants q_{11}, q_{12}, q_{13} and q_{44}. The two alums are miscible in all proportions (Klug & Alexander, 1940) and thus form a continuous series of mixed crystals [$(NH_4)_{1-x}K_xAl(SO_4)_2.12H_2O$].

2. Experimental

For the photoelastic studies, the mixed alum crystals have been grown from solution employing the evaporation technique at constant temperature (300K). Compositions of randomly selected crystals of different compositions have been verified by estimating the nitrogen content, employing the colorimetric analysis.

Bansigir & Iyengar (1958) proposed certain modifications to Filon's spectrometer method. Krishna Rao & Kirshna Murty (1966) employed this method for studying the temperature dependence of the photoelastic behaviour of some alkali halide and MgO crystals. We have also followed the same experimental procedure. The specimen in this method should be in the form of a rectangular prism. As such, the grown crystals of suitable dimensions were properly cut and polished.

Table 1. *Birefringence in T_h-class cubic crystals for various combinations of direction of applied stress and of observation*

Direction of stress	Direction of observation	Birefringence proportional to	Brewster constant
[001]	[100]	$(q_{11}-q_{12})$	$C_{\lambda 1}$
[001]	[010]	$(q_{11}-q_{13})$	$C_{\lambda 2}$
[111]	[$\bar{2}$11]	q_{44}	$C_{\lambda 3}$

The refractive index of different-composition crystals has been determined by the spectrometer method at 4358, 4916, 5461 and 5791 Å employing the immersion technique. The required values of refractive index in the wavelength range covered (4800–6200 Å) for each composition have been derived from these by drawing dispersion curves.

3. Method of calculation

Bhagvantam (1942) noted that the phenomenological theory of photoelasticity given by Pockels (1889, 1906) needed certain corrections. Accordingly, the birefringence produced in the T_h-class crystals on applying compressional stress along different crystallographic directions is as shown in Table 1.

As has been discussed in detail by Krishna Rao & Krishna Murty (1966), the present experimental arrangement is one for determining the Brewster constant C_λ. Combining the experimental values of Brewster constants $C_{\lambda 1}$, $C_{\lambda 2}$ and $C_{\lambda 3}$ with refractive-index data, the various stress–optical constants are calculated by the following formulae.

$$(q_{11}-q_{12})=\frac{2C_{\lambda 1}}{n^3} \qquad (1)$$

$$(q_{11}-q_{13})=\frac{2C_{\lambda 2}}{n^3} \qquad (2)$$

$$q_{44}=\frac{2C_{\lambda 3}}{n^3}, \qquad (3)$$

where n is the refractive index of the crystal in the

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Table 2. *Values of elastic constants* $(c_{11}-c_{12})$ *and* c_{44} *(in units of* $10^{10}\ Nm^{-2}$*) and refractive index (at* 5900 Å*) for* $[(NH_4)_{1-x}K_xAl(SO_4)_2.12H_2O]$ *crystals used in the present calculation*

x	n	$(c_{11}-c_{12})$*	c_{44}*
0·0	1·4606	1·440	0·800
0·2	1·4600	1·446	0·808
0·3	1·4593	1·449	0·812
0·4	1·4592	1·452	0·816
0·5	1·4590	1·455	0·820
0·6	1·4586	1·458	0·824
0·7	1·4578	1·461	0·828
0·8	1·4572	1·464	0·832
1·0	1·4559	1·470	0·840

* American Institute of Physics Handbook (1972).

unstrained condition for the wavelength at which the Brewster constant is determined.

The strain–optical constants are then obtained by combining the stress–optical constants with elastic-constant data using the relations given below.

$$(p_{11}-p_{12})=(c_{11}-c_{12})(q_{11}-q_{12}) \quad (4)$$

$$(p_{11}-p_{13})=(c_{11}-c_{12})(q_{11}-q_{13}) \quad (5)$$

$$p_{44}=c_{44}q_{44}. \quad (6)$$

The elastic constants of mixed (Cr–K) alums (Bhagvantam, 1955) and of alkali-halide mixed crystals have been observed to vary almost linearly with composition. In view of this, the elastic constants for the present system have been computed by linear interpolation. The values of refractive index and elastic constants used in the present calculation are contained in Table 2.

Table 3. *Stress–optical constants (in units of* $10^{-12}m^2N^{-1}$*) and strain–optical constants for* $[(NH_4)_{1-x}K_xAl(SO_4)_2.12H_2O]$ *crystals at* 5900 Å

x	$-(q_{11}-q_{12})$	$-(q_{11}-q_{13})$	$-q_{44}$	$-(p_{11}-p_{12})$	$-(p_{11}-p_{13})$	$-p_{44}$
0·0	5·90	5·21	1·12	0·085	0·075	0·009
0·2	5·53	5·00	0·85	0·080	0·072	0·007
0·3	5·38	4·63	0·75	0·078	0·067	0·006
0·4	5·05	4·43	0·59	0·073	0·064	0·005
0·5	4·99	4·41	0·51	0·073	0·064	0·004
0·6	4·98	4·40	0·51	0·073	0·064	0·004
0·7	5·07	4·46	0·55	0·074	0·065	0·005
0·8	4·86	4·39	0·60	0·071	0·064	0·005
1·0	5·13	4·67	0·64	0·075	0·069	0·005

4. Results and discussion

The calculated values of stress– and strain–optical constants for different compositions at 5900 Å are presented in Table 3. It is seen from this table that these constants vary nonlinearly with composition. The maximum deviation from linearity occurs at around equimolar concentration. The value of these constants for mixed crystals of any composition is less than that expected if the variation were linear. This behaviour has also been observed at all other wavelengths. The same has also been observed by Ethiraj (1975) for the KCl–KBr system.

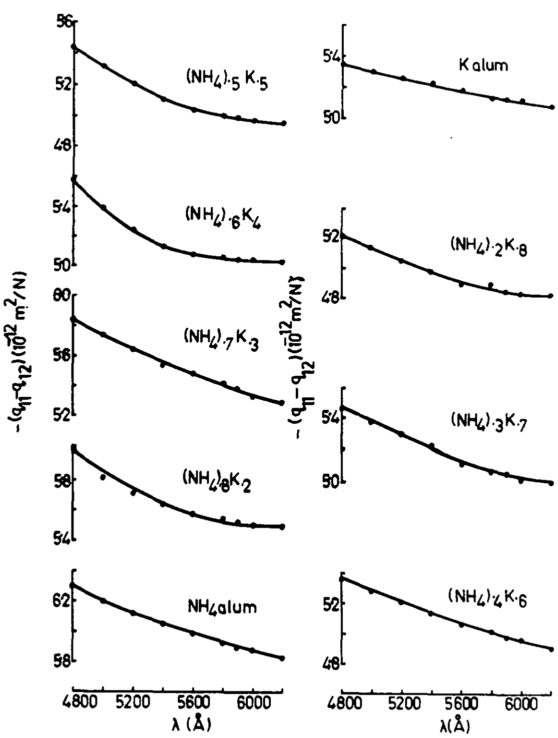

Fig. 1. $(q_{11}-q_{12})$ *vs* wavelength.

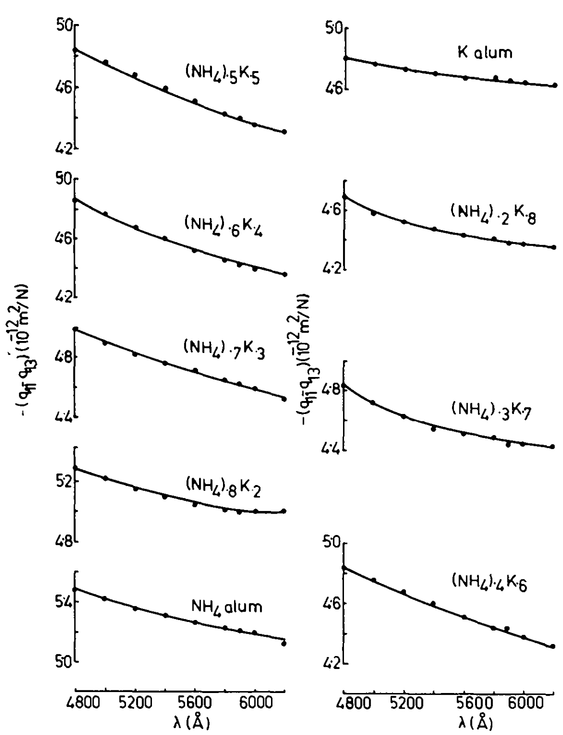

Fig. 2. $(q_{11}-q_{13})$ *vs* wavelength.

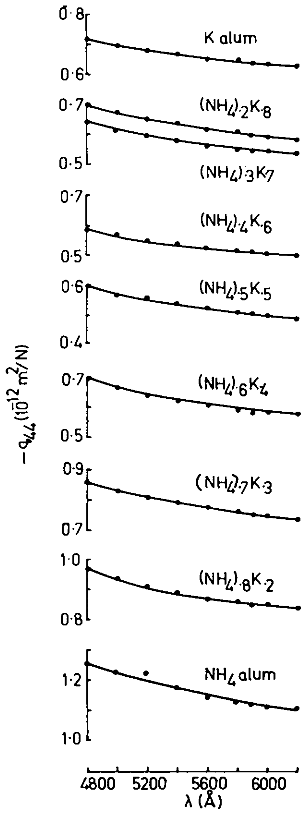

Fig. 3. q_{44} *vs* wavelength.

Many properties of mixed crystals have been observed to vary nonlinearly with composition. In connection with the present studies we determined density and refractive index and found that they varied nonlinearly with composition (Sharma, 1982). We have also measured microhardness (Sharma & Bansigir, 1982) of these crystals and it is also seen to vary nonlinearly. This nonlinearity has also been observed in other mixed crystals.

The wavelength dependence of the stress–optical constants for all compositions studied is shown in Figs. 1–3. The percentage dispersions in $(q_{11}-q_{12})$, $(q_{11}-q_{13})$ and q_{44}, defined by the expression

$$\frac{x_{6200}-x_{4800}}{x_{5500}}\times 100,$$

where x denotes the value of $-(q_{11}-q_{12})$, $-(q_{11}-q_{13})$ and $-q_{44}$, respectively, at the corresponding wavelength, have been calculated for all compositions. Again, the variation of these three constants with composition is found to be nonlinear with largest deviation from nonlinearity occurring at around equimolar composition. For a given composition the percentage dispersion is found to be largest in q_{44} and smallest in $(q_{11}-q_{13})$.

Since the variation of elastic constants with composition is much less than that of stress–optical constants, the strain–optical constants also exhibit similar wavelength dependence to that of stress–optical constants.

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